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Noauthor, Risø

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Some open Questions Concerning the Neutral-Shielding Model of a Fuelling Pellet

C. T. Chang

Risø National Laboratory, DK-4000 Roskilde, Denmark
February 1986

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SOME OPEN QUESTIONS CONCERNING THE NEUTRAL-SHIELDING MODEL OF A FUELLING PELLET

C.T. Chang
Risø National Laboratory - Association Euratom

Abstract. To obtain a better understanding of the implications and assumptions of the idealized neutral shielding model of Parks and Turnbull, the model is reformulated in a self consistent way. Due to the uncertainty of the actual ablation process occuring at the pellet surface, alternative boundary conditions are proposed. Their effect on the pellet ablation rate and the state of the ablated flow are examined by numerical analyses. The results show that the ablation rate is not sensitively affected but the ablatant state is markedly influenced by the boundary condition at the pellet surface. In particular, an increase of the energy flux received at the pellet surface by a factor of four hardly affect the ablation rate but changes the temperature and the density of the ablatant at the pellet surface by three orders of magnitude. Based on these obtained results, it is concluded that the idealized ablation model is adequate when pellet injection is used to fuel a plasma but requires modification when it is used to probe plasma properties and discharge conditions.

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CONTENTS

1. INTRODUCTION

Injection of pellets made of hydrogen isotopes has recently received considerable attention among the controlled nuclear fusion communities [1]. Through the interaction of the injected pellet with the background plasma, increase of plasma density, improvement of the energy confinement time as well as modification of the plasma state and discharge conditions were reported. The experimentally observed pellet penetration depth and the fuel deposition profile were often compared with those predicted by the "Neutral Shielding" model of Parks and Turnbull [2], reasonable agreements were claimed.

On account of the idealization and some implicit assumptions used in the model, a real understanding of its implications and limitations, however, is still lacking. The object of the present paper, accordingly, is to obtain a better understanding and to offer some plausible explanations regarding a few open questions concerned with the model.

The paper is divided into four sections. In Section 2, the model of Ref. [2] is first reformulated in a more self-consistent way. The model considered then is the ablation of a spherical hydrogen pellet subjected to the uniform bombardment of monoenergetic electrons. These electrons are taken to have the same energy and particle flux as that of plasma electrons with a Maxwellian energy distribution but having an energy of $2T_e$.

Following some explicitly stated assumptions, the governing equations describing the ablated flow are presented. In view of the uncertainty of the actual ablation process occuring at the pellet surface, alternative boundary conditions are proposed. These are an ideal energy absorption, a sublimation, and a dynamic phase transition process.

Since both the ablation process and the input parameters of pellet radius, r_p and electron temperature, T_e and density, n_e could affect the solution of the governing equations, their influence on the pellet ablation rate and the state of the ablated flow are studied through numerical analyses. In Section 3, results of computational works are presented and compared.

Finally, in Section 4, after a brief summary, discussions of the obtained results are given. Plausible explanations concerning some open questions associated with the model of Parks and Turnbull are offered. It is concluded that when pellet ablation rate is of main concern, their model, based on the idealization of the energy flux at the pellet surface $q_p \approx o$, is adequate for large product $n_e r_p$ and not a too high electron temperature, T_e. However, when pellet injection is used as a diagnotic means of probing the plasma properties and discharge conditions, further modifications and refinements of present ablation models might be necessary.

2. ANALYTICAL MODEL

In order to formulate a self-consistent model, we shall consider the ablation of a spherical hydrogen (or its isotopes) pellet in a homogeneous unmagnetized plasma. The ion temperature is taken below 10 keV.

Ablation of the pellet then is caused mainly by electron collisions [3]. As a further simplification, we shall neglect the electron energy distribution and consider the ablation of the pellet as caused by uniform bombardment of electron beams. In view of the fact that energetic electrons in the tail of the distribution is more effective causing the ablation, we shall consider the beams to have the same energy flux and particle flux as that of a plasma having Maxwellian electron energy distributions. The energy flux q_o and the energy E_o of these equivalent beams then are given respectively by

$$q_o = n_e \left(\frac{T_e}{2\pi m_e}\right)^{1/2} 2T_e \; , \tag{1}$$

$$E_o = 2T_e \; . \tag{2}$$

In the above and also in the subsequent treatment, temperature is expressed in energy units.

Due to the low binding energy of solid hydrogen and its isotopes, after the introducing of the pellet into a hot plasma, a dense cloud forms almost immediately around the pellet [4]. This cloud then acts as a stopping medium causing the [illegible]ration of the energy and the attenuation of the energy flux of the incoming electrons.

The attenuation of the incoming electron energy flux, q along its trajectory can be considered as a superposition of the depletion of the particle flux, $\Phi(s)$ caused by back scattering and the attenuation of the particle energy E(s) by the slowing-down process, i.e.

$$\frac{dq}{q(s)} = \frac{d\Phi}{\Phi(s)} + \frac{dE}{E(s)} \; . \tag{3}$$

The scattering of the particle flux Φ and the attenuation of the particle energy E are related through the total scattering cross section $\sigma_T(E)$ and the loss function L(E) by

$$\frac{d\Phi}{ds} = \Phi(s) n(s) \sigma_T \left[E(s)\right] \; , \tag{4}$$

$$\frac{dE}{ds} = n(s) L \left[E(s)\right] \; , \tag{5}$$

respectively.

When the energy flux, q_p, at the pellet surface and the heat of the ablation process H_a are known, the mass ablation rate, G, of the pellet can be found from the law of the energy conservation, thus

$$q_p \; 4\pi r_p^2 = G \; H_a \; , \tag{6}$$

using Eqs. (3)-(5), the energy flux q_p at the pellet surface can be related to its value in the unperturbed plasma, q_o by

$$q_p/q_o = (E_p/E_o) \; \exp \left[- \int_{E_p}^{E_o} \frac{\sigma_T(E)}{L(E)} \, dE \right] \; , \tag{7}$$

The problem of the pellet life time or its ablation rate, therefore, reduces to the evaluation of the degraded energy E_p of the hot incoming electrons at the pellet surface.

On account of the variation of the particle density n(s) of the ablatant in the cloud, E_p can be determined only through a study of the hydrodynamics of the expansion process. This is the essential difference of the present problem from the classical treatment of stopping power problems in solids.

2.1. Governing equations

Assuming a steady spherical expansion of the ablated flow and taking the ablated vapor (the ablatant) as a nonconducting, inviscous ideal gas of molecular mass, m, the governing equations of the flow are described by

$$p = \frac{\rho}{m} T \quad , \tag{8}$$

$$G = 4\pi r^2 \rho v \; , \tag{9}$$

$$\rho v \frac{dv}{dr} = - \frac{dp}{dr} \quad , \tag{10}$$

$$\frac{G}{4\pi r^2}\frac{d}{dr}\left(\frac{\gamma}{\gamma-1}T+\frac{v^2}{2}\right) = Q_v(r) \ . \tag{11}$$

In the above $Q_v(r)$ is the volumetric energy source. Through an elimination process, we can reduce the above system of equations to two coupled equations describing the thermal and kinetic energy of the ablated flow, thus

$$(mv^2-T)\frac{dv}{dr} + v\frac{dT}{dr} = 2\frac{vT}{r} \ , \tag{12}$$

$$(mv^2-\gamma T)\frac{dv}{dr} = 2\gamma\frac{vT}{r} - (\gamma-1)\frac{m}{\rho}Q_v(r) \ . \tag{13}$$

We shall now identify the volumetric source $Q_v(r)$ as that part of the attenuated incoming electron energy flux which causes the heating and expansion of the ablatant only, i.e.

$$Q_v(r) = \frac{d}{dr} fq = f\frac{dq}{dr} \ . \tag{14}$$

As a further simplification, we have taken the reduction factor, f as a constant ($\simeq$ 0.6 [2]). This implies that f is an average value, representing all the inelastic processes taking place within the cloud.

Introducing a total energy flux attenuation cross section defined by

$$\Lambda(E) = L(E)/E + \sigma_T(E) \ , \tag{15}$$

we can write

$$\frac{dq}{dr} = \frac{\rho}{m}\Lambda(E)q \ , \tag{16}$$

$$\frac{dE}{dr} = \frac{\rho}{m}L(E) \ , \tag{17}$$

For electrons interact with a molecular hydrogen gas, L(E) and $\sigma_T(E)$ are given by [5], [2]

$$L(E) = [2.35 \times 10^{14} + 4 \times 10^{11} E + 2 \times 10^{17} E^{-2}]^{-1} , \qquad (18)$$

$$\sigma_T(E) = \begin{cases} 1.13 \times 10^{-14} E^{-1} , & (E < 100 \text{ eV}) \\ 8.8 \times 10^{-13} E^{-1.7'} - 1.62 \times 10^{-12} E^{-1.932} , & (E > 100 \text{ eV}) \end{cases} \qquad (19)$$

In Eqs. (18) and (19), E is in eV, $\sigma_T(E)$ in cm^2 and L(E) in $eV - cm^2$.

Using the equation of state and Eq. (14), we can write Eq. (13) alternatively as

$$(mv^2 - \gamma T) \frac{dv}{dr} = \frac{m}{\rho} [2 \frac{\gamma}{r} pv - (\gamma - 1) f \frac{dq}{dr}] . \qquad (20)$$

In the above equation, the first term on the left represents the effect of expansion, whereas the second term that of heating. When we expect a continuos expansion process*, (i.e. $\frac{dv}{dr} > 0$ everywhere), we observe that the flow must be subsonic when heating dominates the expansion process, e.g. near the pellet surface. On the otherhand when expansion dominates the heating process, e.g. far downstream from the pellet surface, the flow must be supersonic. The flow, in general, is transonic in nature. At the sonic radius of expansion r_*, there is a nearly equipartion of the kinetic and the thermal energy of the flow; Eq. (13) becomes singular at the sonic radius.

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* We like to remark here that a continuos flow is a reasonable expectation; an alternative case is possible, where a weak shock occurs at the sonic radius, r_*. The flow then is subsonic everywhere; except it becomes sonic at the singular point.

2.2. Boundary conditions

a) Boundary conditions at the ablated cloud boundary

To ensure a continuous expansion process in their previous treatment of the model [2], Parks and Turnbull has taken

$$p \to o \quad \text{as} \quad r \to \infty . \tag{20}$$

We like to remark here that Eq. (20) actually is a compatability requirement, and is not a boundary condition in the mathematical sense.

Since Eq. (20) corresponds to an expansion into a vacuum background, it is not consistent with a finite energy input. As can be seen from Eq. (10), a necessary condition for $\frac{dv}{dr} > o$ everywhere is that $\frac{dp}{dr} < o$. We expect that a more realistic condition for the existence of a steady expansion should be the pressure balance at the cloud boundary $r = r_P$. This implies that if the pressure of the ablatant falls below that in the ambient plasma, a shock is present. When we are only interested in the pellet ablation rate, we may ignore the shock, so long as it is presented far away from the energy absorbing region.

At the cloud boundary, the energy and the energy flux of the incoming electrons must correspond to their values of the ambient plasma, consequently

$$\left.\begin{array}{l} E \to E_o \\ q \to q_o \end{array}\right\} \text{ when } r \to r_P . \tag{21}$$

b) Boundary conditions at the pellet surface

Due to the lack of knowledge of the actual phenomenon of the ablation process, the boundary conditions at the pellet surface are rather uncertain. We shall, therefore, present some alternative considerations in terms of the physical mechanism of the ablation process.

i) Ideal energy absorption

If the ablation cloud is viewed as a perfect energy absorbing medium, we may idealize the situation and take the energy flux at the pellet surface to be vanishingly small, $q_p \approx o$.

The conservation of energy at the pellet surface Eq. (6), then implies for a finite ablation rate, G the heat of ablation

$$H_a \approx o \ . \tag{22}$$

where H_a is given explicity by

$$H_a = \frac{\gamma}{\gamma-1}\,\frac{T_v}{m} + \frac{v_v^2}{2} + \frac{\varepsilon}{m} \qquad . \tag{23}$$

In Eq. (23) the first two terms represent the specific enthalpy and kinetic energy of the ablatant at the pellet surface; the last term represents the energy corresponding to the phase transition process. When the ablatant is in a molecular state, we can take ε as the sublimation energy of solid hydrogen ($\approx 10^{-2}$ eV).

When we neglect the sublimation Energy ε and anticipate a subsonic flow at the pellet surface, Eq. (22) can be satisfied when T_v is vanishingly small.

Following these considerations, we can take the boundary conditions at the pellet surface as

$$\left.\begin{matrix} q_p = o \\ T_v = o \end{matrix}\right\} \text{ at } r = r_p \ . \tag{24}$$

We like to remark here that Eq. (24) is consistent only when we take $\varepsilon = o$.

ii) Ablation as a sublimation process

If the ablation of the pellet is taken to be a sublimation process, then clearly ε must be finite. Denoting $\dot{\phi}_p$ as the ablated particle flux, the energy flux q_p at the pellet surface must satisfy the requirement that

$$q_p \gtrsim q_s \tag{25}$$

where $q_s = \dot{\phi}_p\,\varepsilon$ is the energy flux corresponding to the sublimation process.

iii) Ablation as a dynamic phase transition

Previous investigation [7], indicated that the pellet life time in a reasonable hot and dense plasma is much shorter than the time required for the sublimation process. This fact implies that the ablation of the pellet is caused by the propagation of an evaporation front, i.e. a dynamic phase transition process.

In terms of a parameter χ defined as

$$\chi \equiv \frac{q_p - q_s}{\dot{\phi}_p T_v} = \frac{\gamma}{\gamma-1}\,\frac{h_v + \dfrac{v_v^2}{2}}{h_v} , \tag{26}$$

where $h_v = \dfrac{\gamma}{\gamma-1}\,\dfrac{T_v}{m}$ is the enthaply per ablated particle leaving the pellet. A necessary and sufficient condition for the existence of a dynamic phase transition then requires that χ satisfies the following condition

$$\frac{\gamma}{2}\left(\frac{\gamma+1}{\gamma-1}\right) > \chi > \frac{\gamma}{\gamma-1} . \tag{27}$$

The lower limit implies that $\frac{v_v^2}{2} \ll h_v$, a necessity of the flow being subsonic at the pellet surface. The upper limit simply means a definite propagation speed of the evaporation front.

2.3. Normalization

In view of the presence of a singularity in Eq. (13), at the sonic radius, r_*, it will be appropriate to normalize all variables in the governing equations by their corresponding values at r_*. Introducing $v = v_* v'$, $T = T_* T'$, $\Lambda = \Lambda_* \Lambda'$ .... etc. and using the requirement of mass conservation at the sonic radius, Eq. (13) in its dimensionless form becomes.

$$\frac{dv'}{dr'} = 2v'T' \frac{\left[\frac{A_* \lambda_*}{2} \frac{\Lambda' q'}{v'T'} - \frac{1}{r'}\right]}{(T'-v'^2)} , \tag{28}$$

$$\text{where } A_* = \left(\frac{\gamma-1}{\gamma}\right) 4\pi r_*^2 \left(\frac{m}{G}\right) \frac{fq_*}{T_*} , \tag{29}$$

$$\text{and } \lambda_* = \rho_* \Lambda_* r_*/m . \tag{30}$$

Since at the sonic radius, all variables become unity, the definiteness of $\left(\frac{dv'}{dr'}\right)_{r'=1}$ requires that

$$A_* \lambda_*/2 = 1 . \tag{31}$$

Substituting Eqs. (29) and (30) into the above we obtain

$$4\pi r_*^2 (fq_*) = 2\left(\frac{G}{\lambda_*}\right) \frac{\gamma}{\gamma-1} \frac{T_*}{m} . \tag{32}$$

We recall that $\frac{\gamma}{\gamma-1}\frac{T_*}{m}$ is the enthalpy of the of the ablatant at r_*. Since at the sonic radius, r_*, there is a nearly equipartion of the thermal and kinetic energy of the flow, Eq. (32) simply can be interpreted as the energy conservation requirement at the sonic radius. This implies the parameter λ_* must be near unity.

Using Eq. (31), the system of governing equations finally can be written in their equivalent dimensionless forms, thus

$$\frac{dv}{dr} = 2\,\frac{VT}{(T-V^2)}\left[\frac{\Lambda q}{TV} - \frac{1}{r}\right] , \tag{33}$$

$$\frac{dT}{dr} = 2\,\frac{\Lambda q}{v} - (\gamma-1)v\,\frac{dv}{dr} , \tag{34}$$

$$\frac{dq}{dr} = \lambda_*\,\frac{\Lambda q}{r^2 v} , \tag{35}$$

$$\frac{dE}{dr} = 2\lambda_*\left[\frac{L(EE_*)}{E_*\Lambda_*}\right]\frac{1}{r^2 v} , \tag{36}$$

$$\rho v r^2 = 1 , \tag{37}$$

$$p = \rho T . \tag{38}$$

For the simplification of notation, we have dropped the "," notation of all the variables appeared in the above equations. One notices that the system of equations contains two parameters E_* and λ_*. Since their values have to be chosen in accordance with the boundary conditions specified at the pellet surface and the cloud boundary, we may consider them as the eigen values of the governing equations. To faciliate computational works, we have shown that it is more convenient to replace λ_* by a parameter K, defined as

$$K = \lambda_* + \left(\frac{dr}{dr}\right)_{r=1} . \tag{39}$$

For $\left(\frac{dv}{dr}\right)_{r=1} = Z_S$ to be positive and definite, K has to be chosen within the open interval (0, 1) [8]. In view of the fact that H_a, the total heat of ablation cannot be less than h, the enthalpy of the ablatant, Eq. (23), it was shown previously [6] that the choice of K is further restricted to the requirement that

$$\hat{T} < \hat{q} ,$$

where $\hat{T} \equiv T_v/T_*$ and $\hat{q} \equiv q_p/q_*$.

T_v and q_p are the ablatant temperature and electron energy flux at the pellet surface; T_* and q_* are their corresponding values at the sonic radius.

In terms of the dimensionless variables, the three alternative boundary conditions at the pellet surface

$$\hat{r} \equiv r_p/r_* , \tag{40}$$

now can be written as the following:

i) Ideal energy absorption process

$$\hat{q} < \hat{q}_c , \hat{T} \approx \hat{q} , \tag{41}$$

where $\hat{q}_c$ is a smallest positive number for which $\hat{T} \approx \hat{q}$.

ii) Sublimation process

$$X \approx 0 . \tag{42}$$

We like to remark here that this is a necessary requirement but not a sufficient description of the sublimation process.

iii) Dynamic phase transition

$$4.2 > \chi > 3.5 \text{ , for } \quad \gamma = 7/5 \text{ .} \tag{43}$$

At the cloud boundary, the boundary condition becomes

$$E \to \tilde{E} \equiv E_o/E_* \quad , \quad q \to \tilde{q} \equiv q_o/q_* , \tag{44}$$

when $r \to r_P/r_*$.

2.4. Flow parameters at the sonic radius and the scaling law of the ablation rate

Since all the flow variables are normalized with respect to their corresponding values at the sonic radius, it is necessary to determine their values at the sonic radius to obtain their physical values. At the sonic radius, we have five available equations. These are the flow velocity being sonic, the equation of the state, the conservation of mass, the definition of λ_*, Eq. (30) and the restrictive condition, Eq. (32). These five equations are sufficient to solve for the ablation state and the mass ablation rate G_* in terms of E_*, q_* and r_*, thus

$$\frac{\gamma T_*}{m} = \left[\frac{(\gamma-1) f q_* r_* \lambda_*}{2m}\right]^{2/3} , \tag{45}$$

$$\gamma p_* = \left[\frac{m}{r_* \Lambda_*}\right]^{1/3} \left[\frac{(\gamma-1) f q_*}{2}\right]^{2/3} \lambda_* \quad , \tag{46}$$

and $$G = 4\pi\lambda_* \, m^{2/3} \left[\frac{(\gamma-1) f q_*}{2\Lambda_*^2}\right]^{1/3} r_*^{4/3} \quad . \tag{47}$$

One must recall that λ_* and Λ_* are functions of E_*.

Using the definitions of $\tilde{q}$, Eq. (44) and $\hat{r}$, Eq. (40), the particle ablation rate, $\dot{N}_p$, can be written as

$$\dot{N}_p = 2\lambda_*(2\pi)^{5/6} \left[(\gamma-1)f \frac{m_e}{m} / (\hat{r}^4\tilde{q})\right]^{1/3} (r_p^4 n_e)^{1/3} \left(\frac{T_e}{m_e}\right)^{1/2} \left(\frac{1}{\Lambda_*}\right)^{2/3}, \tag{48}$$

where $\Lambda_* \equiv \Lambda_* (E) = \Lambda\ (2T_e/E)$.

We like to remark here that Eq. (48) is derived from a mere consideration of the conservation of mass and the definitiveness of (dv/dr) at the sonic radius.

Furthermore, one observes that the variable $\dot{N}_p/(r_p^4 n_o)^{1/3}$ has the same dimension as that of $\left(\frac{T_e}{m_e}\right)^{1/2} \left(\frac{1}{\Lambda_*}\right)^{2/3}$, this indicates that

$$\dot{N}_p/(r_p^4 n_e)^{1/3} = f(T_e) \text{ only.} \tag{49}$$

The different boundary conditions, therefore, can only affect the numerical values of λ_*, $\hat{r}$, E and $\tilde{q}$.

2.5. Method of solution

Applying the L'Hopitâl's rule, the derivative $Z_s \equiv \left(\frac{dv}{dr}\right)_{r=1}$ can be determined and is found to be a function of E_* and λ_* (or equivalently K). Once E_* and K are assigned all the derivatives of Eqs. (33)-(36) are known. We may then start the integration at the sonic radius r = 1, using the boundary condition, Eq. (41) or Eq. (42), or Eq. (43) to locate the pellet position, $\hat{r}$.

Finally, the cloud boundary can be located by applying Eq. (44). In principle, the system of equations has to be solved iteratively by choicing proper values of E_* and λ_* with respect to given pellet radius, r_p and plasma state, E_o, q_o or its equivalent T_e and n_e.

We like to mention here that in view of the method used in solving the problem, it is formulated as an initial value problem mathematically. The "boundary conditions" at the pellet surface and the cloud boundary should be viewed as constraints, imposed for the proper choice of the eigen values E_* and λ_*.

3. NUMERICAL ANALYSIS

In principle, the choice of E_* and K, thus the solution of the system of equations, Eqs. (33)-(36), depends not only on the particular boundary conditions used but also on the input parameters; pellet radius, r_p and plasma state, n_e and T_e.

As E_*, the energy of the incident electrons at the sonic radius is expected to depend mostly on the ambient plasma electron temperature, T_e, we shall, for convenience, replace the input parameters by E_*, n_e and r_p instead.

An additional parameter requires some consideration. Previous works showed that on account of the rapid attenuation of the incoming electron energy, the energy flux drops steeply near the pellet surface. On the otherhand, the density of the ablatant decreases while its temperature rises rapidly for a slight increase of the radial position away from the pellet surface. This observation suggests that a slight variation of a cut-off value $\hat{q}_c$ might not affect the pellet location, but could cause substantial variations in the ablatant state. Intuitively, one would expect that a higher value of q_p (or $\hat{q}_c$) could increase either the ablation rate or the total heat of ablation, H_a. In view of the idealization of perfect energy absorption, i.e. $q_p = o$, this arbitrainess of q_p is absent. On the other hand, from the very definition of χ, Eq. (26), its value, clearly, will be affected by the choice of q_p, or its equivalent $\hat{q}_c$.

As a preliminary study to find the effect of the input parameters on the choice of the eigen value K, we have taken the same value of E_* but varied n_e and r_p. Computations were performed by using the same boundary condition at the cloud boundary, Eq. (44), and the requirement of

$$\hat{T} > o \quad \text{and} \quad \hat{q} > o \; .$$

The effect of the boundary condition at the pellet surface were then studied by examining the variation of $\hat{T}$, $\hat{q}$ and χ with respect to the values of K. The results are shown in Figs. 1 and 2 for $E_* = 2.0$ keV, $n_e = 5 \times 10^{13}$ cm^{-3}, $r_p = 0.1$ cm, and $E_* = 2.0$ keV, $n_e = 2 \times 10^{14}$ cm^{-3}, $r_p = 0.25$ cm, respectively. We notice that when the idealized $q_p \simeq o$, Eq. (41), is used as the boundary condition at the pellet surface, indeed, the same value of K is obtained, independent on the particular values of n_e and r_p. In the more realistic case of finite q_p, Eq. (42) or Eq. (43), there is not much difference in the corresponding values of K whether q_p is just sufficient to cause sublimation ($\chi = o$, Eq. (42)), or q_p drives an evaporation front (Eq. (43)). This value of K corresponding to a finite q_p, however, is smaller than the one corresponding to $q_p \simeq o$. The difference becomes less for larger values of $n_e r_p$. This indicates that although the ideal energy absorption process is not physically realistic, the approximation becomes better for large values of $n_e r_p$.

Details of the solutions regarding the ablatant state and the ablation rate corresponding to these alternative ablation processes are given in Table I. In the table, the first row represents result obtained when Eq. (43) is used; the second, Eq. (42) and the last, Eq. (41). One notices that both the sonic radius r_* ($\equiv r_p/\hat{r}$) and the particle ablation rate $\dot{N}_p$ are practically the same in all the three cases considered. The result, therefore, confirms the previous assertion that the pellet ablation rate is not sensitive to the details of the ablation process occuring at the pellet surface [2], [7]. In all the three cases considered, the ablatant temperature, T_v near the pellet surface is very low, while its density, n_v is rather high, particularly for the low density small pellet case.

To study the solution corresponding to a more realistic ablation process, the effect of the pellet radius, r_p, and the plasma electron density n_e on the value of χ was investigated first.

Computations were performed by taking a fixed E_* and a sufficiently small $\hat{q}_c$ (e.g. $\hat{q}_c = 1.0 \times 10^{-5}$), and varying n_e and r_p. The results are shown in Table II for $E_* = 2.0$ keV and $K = 0.29069$.

From the values listed in Table II, it is interesting to note that at various combinations of n_e and r_p, exactly same values of χ and T_* are obtained as long as the product $n_e r_p$ remains the same.

The effect of the cut-off value $\hat{q}_c$ was then investigated by taking the same values of E_* and K as in the previous case but with values of $r_p = 0.1$ cm, and $n_e = 4.90 \times 10^{13}$ cm^{-3} respectively. The result is shown in Fig. 3. It is interesting to note that there exists a limiting value q_{c1}, i.e. for values of $\hat{q}_c <$ $\hat{q}_{c1}$, the solutions are identical in all aspects such as the ablation rate and flow variables. Furthermore, once $\hat{q}_c$ passes beyond $\hat{q}_{c1}$, χ decreases rapidly and approaches the limit of $\chi = 3.5$. This value of χ, incidently, corresponds to the case that the ablatant leaves the pellet at zero kinetic energy. We should like to remark here that the same limiting value $\hat{q}_{c1}$ exists for all combinations of n_e and r_p listed in Table II. This implies that $\hat{q}_c = 1.0 \times 10^{-5}$ chosen previously indeed is sufficiently low and has no influence on the solutions obtained.

One observes from the values tabulated in Fig. 3 that a mere increase of $\hat{q}_c$ by a factor of four, the temperature T_v increases while the density, n_v decreases by three orders of magnitude.

The pellet location, r_p/r_*, and the particle ablation rate $\dot{N}_p$, on the otherhand, are almost the same. The reason that the ablation rate remains almost the same at a higher value of q_p, clearly, is due to the higher heat of ablation, H_a, i.e. the ablatant leaves the pellet at a higher thermal and kinetic energy. The result obtained now indicates that when a more realistic ablation process is considered as the boundary condition, Eq. (42) or Eq. (43) must be supplemented by a proper cut-off value $\hat{q}_c$, to ensure the ablatant leaving the surface not extremely cold and excessively dense. In the case just studied, as can be seen from Fig. 3, $\hat{q}_c > 2.0 \times 10^{-3}$.

Following this indication, taking $\hat{q}_c = 2.0 \times 10^{-3}$ the same E_* and K as previously, we have varied r_p and n_e to find their influence on the ablatant state near the pellet surface. The result is shown in Fig. 4. One observes that when the ablation of the pellet is considered as a dynamic phase transition, the permissible range of $n_e r_p$ is bounded by the lines aa and bb shown in the figure. Specifically, the result shows that when a 1 mm radius pellet is injected into a plasma of $T_e \approx 1.2$ keV and $n_e \approx 5.0 \times 10^{13}$ cm^{-3} the ablatant leaves the pellet surface at $T_v \approx 7.2$ degK and $n_v \approx 2.94 \times 10^{22}$ cm^{-3}. The corresponding values at the sonic radius, r_* are $T_* \approx 2$ eV and $n_* \approx 2.88 \times 10^{18}$ cm^{-3}.

4. SUMMARY AND DISCUSSIONS

As a first step to study the ablation of a hydrogen pellet in a plasma environment, we have considered the ablation of a spherical pellet in a unmagnetized homogeneous plasma with ion temperatur below 10 keV. Ablation then is caused mainly by electron collisions. Further simplication is made by neglecting the energy distribution of electrons by replacing the Maxwellian distributed electrons with a monoenergetic beam of energy 2 T_e and the same energy flux. The ablation process then is studied as a "stopping-power" problem of the degration of the electron energy flux in a dense ablated cloud.

On account of the spatial variation of the density of the ablatant, the attenuation of the electron energy flux is formulated as a hydrodynamic problem of spheric expansion. The governing equations of the flow are shown to be singular at the sonic radius of expansion. When one seeks a continuos solution, the flow becomes transonic in nature. The system of the governing equations are then solved as an initial value problem from the sonic radius with two adjustable parameters; E_*, (the attenuated electron energy at the sonic radius), and λ_*. The latter can be interpreted as related the mass of the stopping medium at the sonic radius.

It was shown that a scaling law of the pellet ablation rate with respect to the plasma state and pellet radius can be derived from a consideration of the mass conservation and the definitiveness of (dv/dr) at the sonic radius alone. The magnitude of the ablation rate, however, has to be determined by solving the governing equations with appropriate boundary conditions.

In view of the uncertainty of the ablation process, alternative boundary conditions are proposed. These are a perfect energy absorption, a sublimation and a dynamic phase transition process respectively. The effect of the boundary conditions on the solutions are then examined through extensive numerical analyses. Among the results obtained, the following are particularly noteworthy.

i) When the energy flux at the pellet surface is vanishingly small, i.e $q_p = o$, Eq. (41), as shown by results in Table I, Figs. 1 and 2, the solution depends only on E_*, thus the plasma temperature, T_e; and is independent on any particular choice of the electron density, n_e and the pellet radius, r_p.

ii) Irrespective of the particular boundary condition at the pellet surface, as shown by results in Table I, the pellet ablation rate is nearly the same for given values of r_p, T_e and n_e.

iii) When a more realistic ablation process is taken, Eq. (42) or Eq. (43), as shown by results in Table II, at various combination of n_e and r_p the solution depends on the product $n_e r_p$ only.

iv) Under a more realistic situation at the pellet surface, namely a finite energy flux q_p, a reasonable warm and not extremely dense ablatant, the attenuation factor $\eta = q_p/q_o$ is limited. For the range of pellet radius and plasma state of interest, $\eta < 10^{-3}$.

As mentioned previously in term of the given parameters of r_p, n_e and T_e, using the present method of solution, the problem has to be solved iteratively. As long as the pellet ablation rate is our main concern, the iterative procedure is much simpler when the ideal energy absorption process, $q_p \approx o$, is taken as the boundary condition at the pellet surface. Before attempting to give some plausible explanations of the above observations, therefore, it should be of interest to examine the validity of this approximation first.

By eliminating the mass ablation rate G between Eq. (6) and Eq. (32), the energy flux at the pellet surface, q_p can be related to that at the sonic radius, q_*. Using the definition of $\hat{r} = r_p/r_*$, we obtain

$$\hat{q} = q_p/q_*$$

$$= (\frac{1}{\hat{r}})^2 \frac{\gamma-1}{2\gamma} \frac{f\lambda_* mH_a}{T_*} \quad ,$$

Using Eq. (45), eliminating T_*, finally we obtain

$$\hat{q} = (\frac{f\lambda_* H_a}{2T_e}) \ [2\pi(\gamma-1)m_e]^{1/3} \ [\frac{m\tilde{q}}{f(\Lambda_* n_e r_p)\hat{r}^2}]^{2/3} \ . \qquad (50)$$

Equation (50) shows that $\hat{q} = o$ if the heat of ablation $H_a = o$. In view of the flow is subsonic at the pellet surface. Eq. (23) shows that $H_a = o$ is a consequence by neglecting the sublimation energy, ε, and taking $T_v = o$. On the otherhand, for any finite small H_a, $\hat{q}$ will be small if $(\Lambda_* n_e r_p)$ is large. Recalling that Λ is a decreasing function of the electron energy for $E_* > 100$ eV, [4] the ideal energy absorption process consequently, might break down for small product of $n_e r_p$ and high plasma temperature.

For a given E_* (thus known Λ_*), when λ_* is chosen to satisfy $\hat{q} = o$ and $\hat{T} = o$ at the pellet surface, Eq. (50) shows that no knowledge of n_e and r_p is required. When a realistic ablation process is considered e.g. Eq. (42) or Eq. (43), q_p or $\hat{q}$ must be finite. Equation (50) shows that $\hat{q}$ depends on the product $n_e r_p$.

The apperance of the product $n_e r_p$ as an important parameter of $\hat{q}$. Equation (50), is worthy of attention. One notices that $n_e r_p$ has the same dimension as the "range" parameter in stopping power terminology. From the results shown in Table II and Fig. 4, one observes that at a given E_*, the product $n_* r_p$ is the same at various combination of n_e and r_p irrespective of the value of the parameter χ. This result together with the identical values of $\hat{r} = r_p/r_*$ and $\hat{E} = E_o/E_*$ yields the same value of $n_* r_*$, the range at the sonic radius at a given incident electron energy E_o.

As the plasma electrons stream through the ablated cloud, most of their energies are absorbed after passing through the sonic radius. Besides, as it was shown in Ref. [4], the stopping power of electrons in a molecular hydrogen gas is not much different from that in a solid hydrogen. This observed insensitiveness of the range parameter, $n_* r_*$, on the value of χ could then possibly explain the fact that for given plasma electron density and pellet radius, the pellet ablation rate is governed mainly by the incident electron energy and is not sensitive to the detailed mechanisms of the ablation process occurring at the pellet surface.

In conclusion, the present analysis showed that when pellet injection is used mainly to fuel a tokamak plasma, the current neutral shielding model, in spite of its idealization, is a valid approximation. On the otherhand, when pellet injection is used to probe plasma properties, discharge conditions, etc, in view of their dependence on the state of the ablatant, further modification and refinement of the model are required.

Table I. Comparison of solutions at $E_* = 2.0$ keV using the alternative boundary conditions (Program PELABNSM/DPT).

B.C.	λ_*	$\hat{r}$	$\hat{q}$	$\tilde{q}$	T_v/T_s	n_v/n_s	T_e, keV	$\dot{N}_p$, sec
a) $n_e = 5.0 \times 10^{13}$ cm^{-3} $r_p = 0.01$ cm								
$\chi = 3.896$	0.98866	0.6753	7.5154^{-3}	1.5514	7.090^{-3}	5.460^{2}	1.2022	4.2416^{22}
$\chi = 0.181$	0.98867	"	7.5109^{-3}	"	7.116^{-3}	5.447^{2}	"	"
$\hat{T} = 5.7135^{-6}$ ($\chi = -4.238^{3}$)	0.99547	0.6746	9.8102^{-6}	1.5554	8.172^{-5}	4.767^{4}	1.2035	4.2753^{22}
b) $n_e = 2.0 \times 10^{14}$ cm^{3} $r_p = 0.25$ cm								
$\chi = 3.987$	0.995153	0.6747	3.5260^{-4}	1.5552	2.346^{-2}	1.431^{2}	1.2034	4.9592^{24}
$\chi = 0.682$	0.995154	"	3.5166^{-4}	"	2.362^{-2}	1.421^{2}	"	"
$\hat{T} = 5.7135^{-6}$ ($\chi = -1.915^{2}$)	0.99547	0.6746	9.8102^{-6}	1.5554	1.473^{-1}	2.279^{1}	1.2035	4.9610^{24}

where $7.5109^{-3} = 7.5109 \times 10^{-3}$, $T_s = 4$ deg. K, $n_s = 3 \times 10^{22}$ cm^{-3}

Table II. Effect of n_e and r_p on the value χ, Eq. (26).

n_e 10^{13} cm^{-3}	n_* 10^{18} cm^{-3}	n_v 10^{24} cm^{-3}	T_* eV	T_v 10^{-6} eV	p_* 10^6 dynes/cm^2	q_* 10^6W/cm^2	q_p 10^4 W/cm^2	$\dot{N}_p$ 10^{23}/s	$\dot{N}_p/(r_p^4 n_e)$ 10^{20}	χ
		r_p = 0.05 cm			$r_p n_*$ = 2.9404 x 10^{17} cm^{-2}					
8.0	5.8808	7.6197	1.6500	4.2235	15.546	11.516	1.8977	4.2678	5.3772	-3.033^2
9.60	"	"	1.8632	4.7693	17.556	13.819	2.2773	4.5352	5.3771	-32.36
(9.856)	"	"	(1.8962)	(4.8537)	(17.866)	14.187	(2.3380)	4.5752	5.3776	(4.111)
10	"	"	1.9146	4.9009	18.040	14.394	2.3722	4.5974	5.3772	23.94
		r_p = 0.10 cm			$r_p n_*$ = 2.9404 × 10^{17} cm^{-2}					
4.0	2.9404	3.8098	1.6500	4.2235	7.7731	5.7578	0.94886	8.5357	5.3772	-3.033^2
4.80	"	"	1.8632	4.7693	8.7778	6.9093	1.1386	9.0705	5.3772	-32.36
(4.928)	"	"	(1.8962)	(4.8537)	(8.9331)	7.0936	(1.1690)	9.1504	5.3776	(4.111)
5.0	"	"	1.9146	4.9009	9.0199	7.1932	1.1861	9.1948	5.3772	23.94
		r_p = 0.20 cm			$r_p n_*$ = 2.9404 × 10^{17} cm^{-2}					
2.0	1.4702	1.9049	1.6500	4.2235	3.8866	2.8789	0.47443	17.071	5.3772	-3.933^2
2.40	"	"	1.8632	4.7693	4.3889	3.4547	0.56932	18.141	5.3772	-32.36
(2.464)	"	"	(1.8962)	(4.8537)	(4.4666)	3.5468	(0.58450)	18.301	5.3777	(4.111)
2.50	"	"	1.9146	4.9009	4.5100	3.5986	0.59304	18.390	5.3773	23.94

E = 2.0 keV
λ = 0.9939807
(K = 0.29069)

B.C.
$\hat{T} < \hat{q}$
$\hat{q}_c = 1.0 \times 10^{-5}$

The following values are the same for all cases,
$\hat{r}$ = 0.67478 , $\hat{q}$ = 1.6480^{-3} , $\hat{T}$ = 2.5597^{-6} , $\hat{E}$ = 6.3139^{-2}
T_e = 1.2032 keV , $\tilde{q}$ = 1.5545 , $\tilde{T}$ = 9.8808 , $\tilde{E}$ = 1.2032

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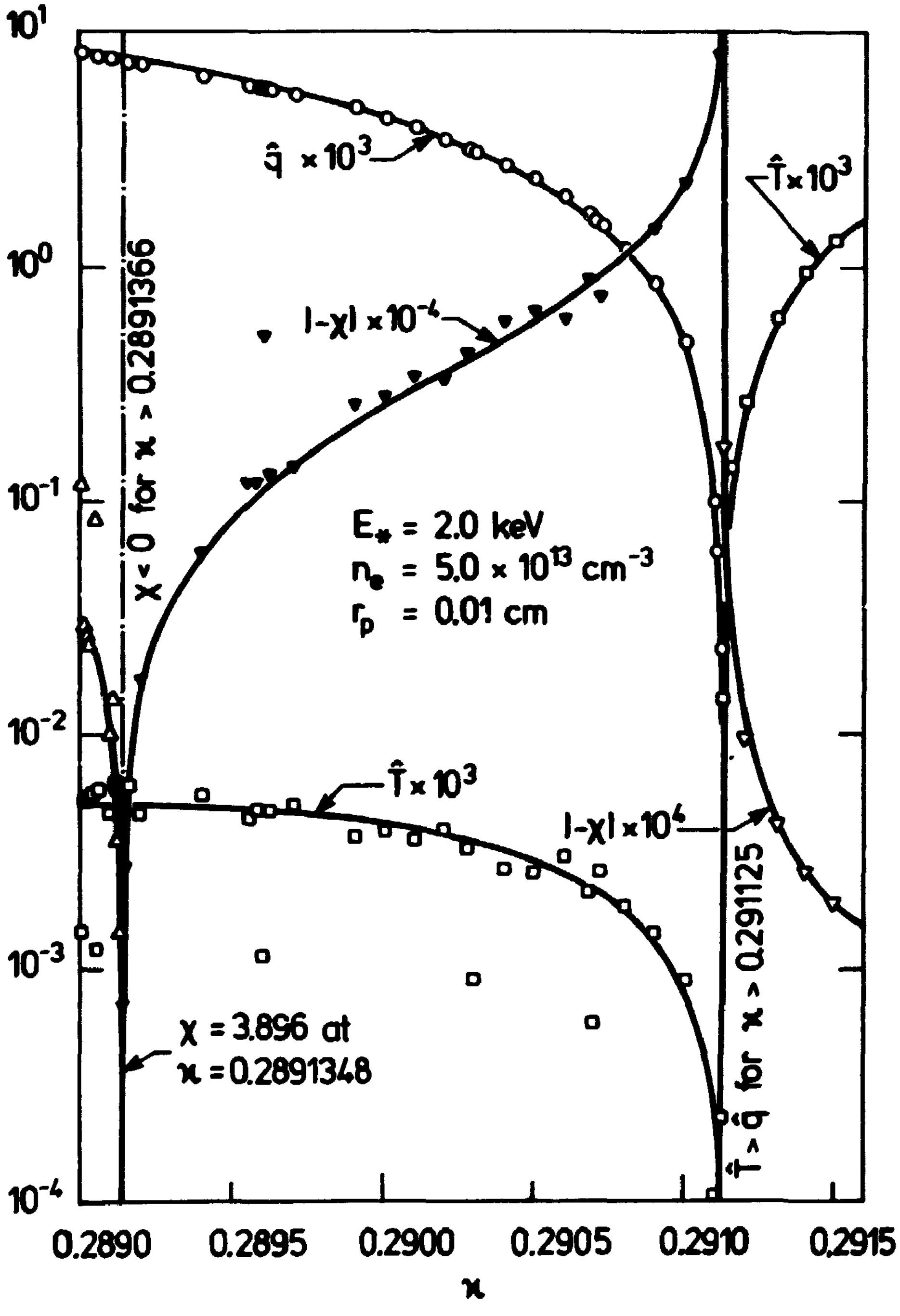

Fig. 1. Variation of $\hat{q}$, $\hat{T}$ and χ with respect to the eigenvalue $\varkappa$. Possible solutions are limited to the left of the line $\hat{T} < \hat{q}$. $E_* = 2.0$ keV, $n_e = 5.0 \times 10^{13}$ cm^{-3}, $r_p = 0.01$ cm.

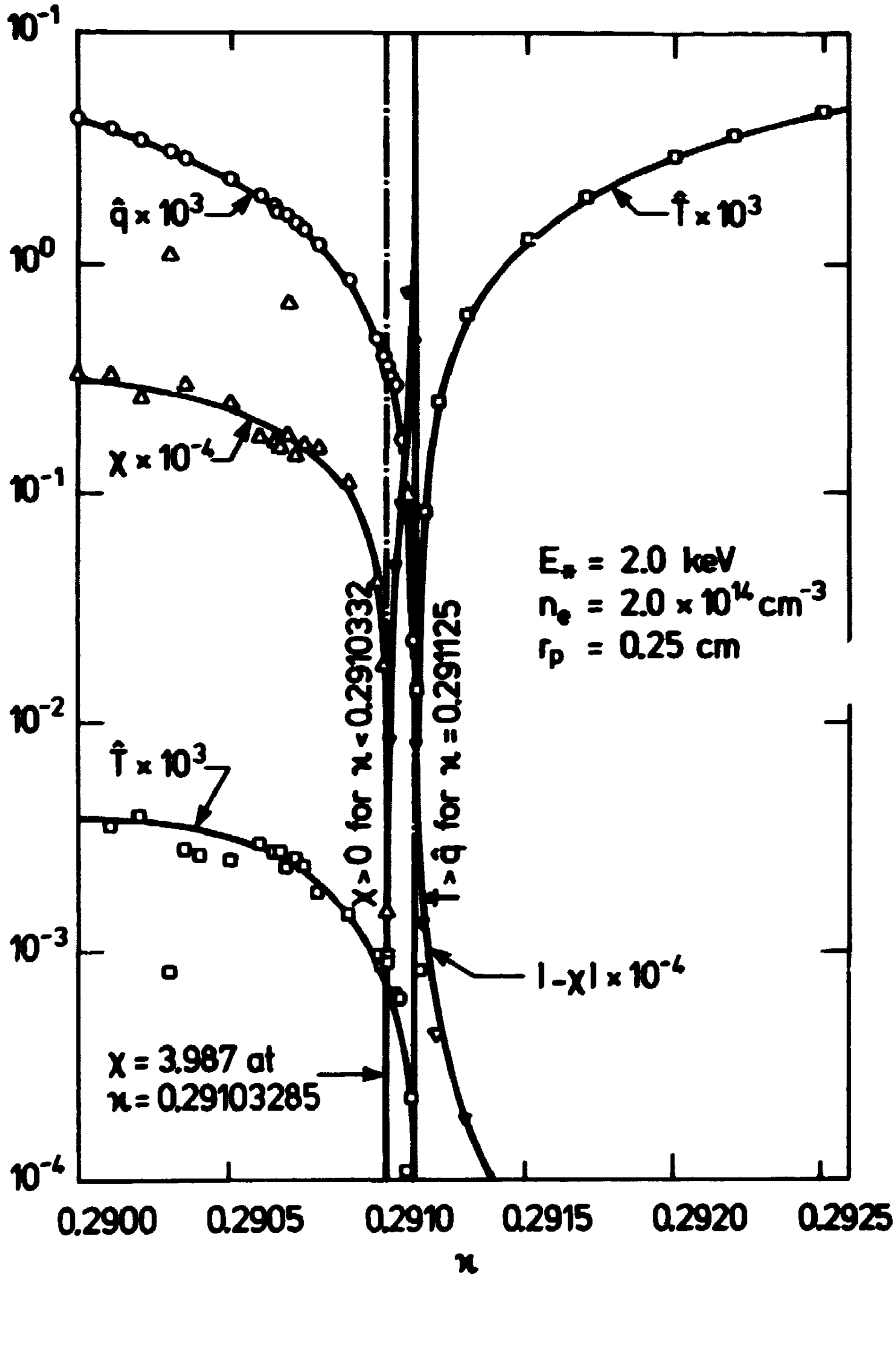

Fig. 2. Variation of $\hat{q}$, $\hat{T}$ and χ with respect to the eigenvalue K. E_* = 2.0 keV, n_e = 2.0×10^{14} cm^{-3}, r_p = 0.25 cm. Notice the line $\hat{T} = \hat{q}$ is located at the same value of K as in Fig. 1.

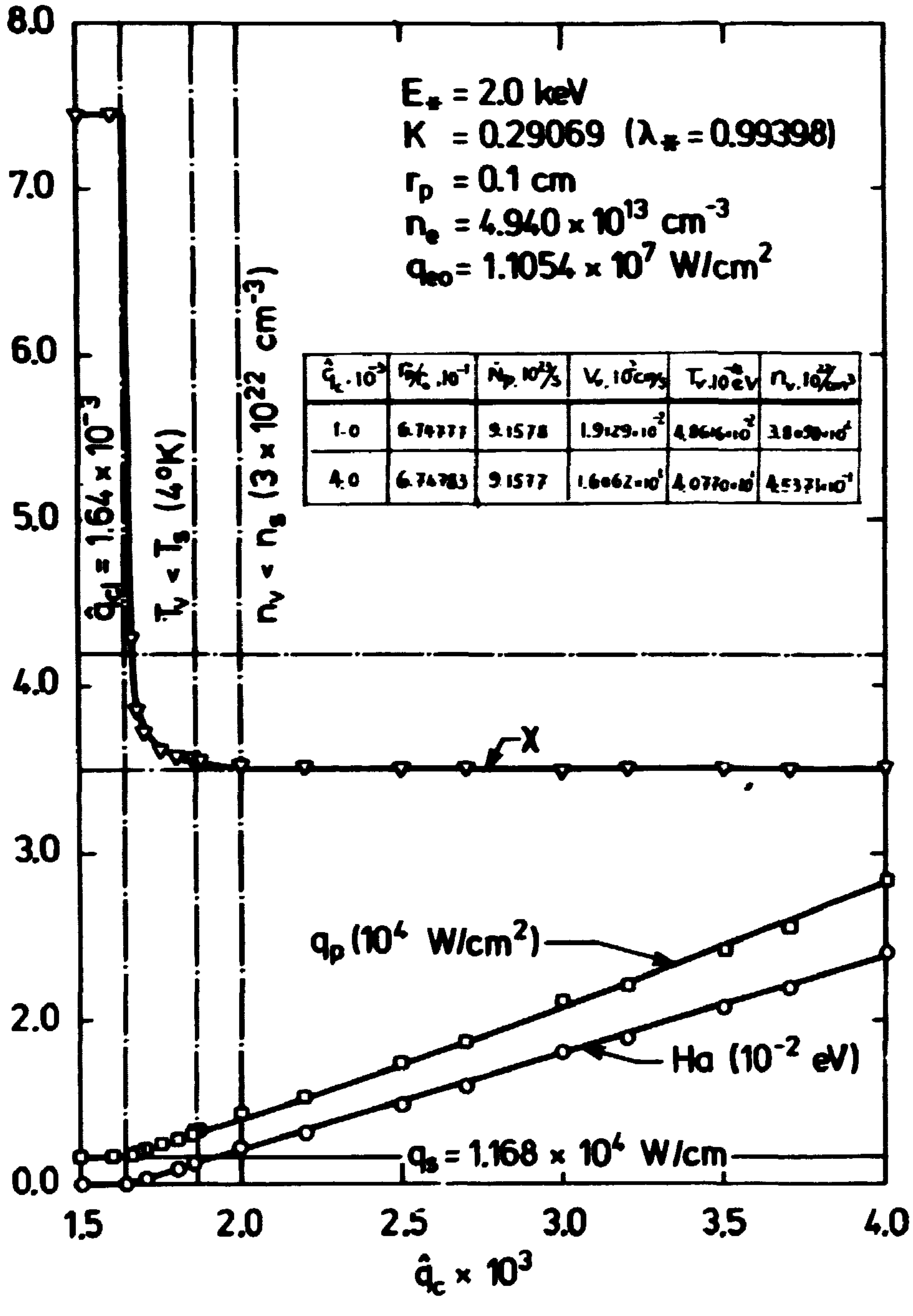

Fig. 3. The effect of the cut-off value $\hat{q}_c$ on the energy flux q_p at the pellet surface and the parameter χ. Notice the existence of a limit value of $\hat{q}_c$ at $\hat{q}_{cl} = 1.64 \times 10^{-3}$.

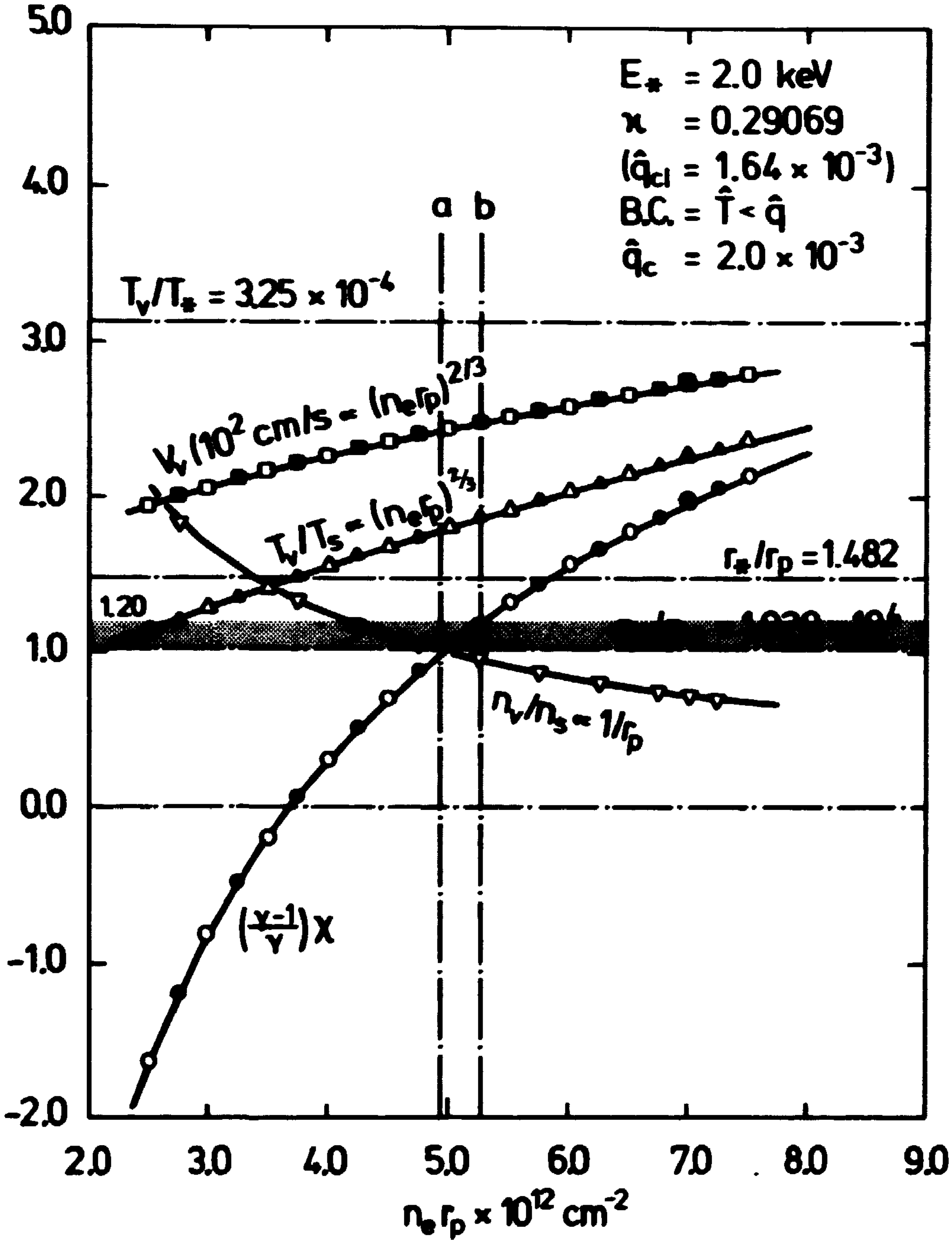

Fig. 4. Variation of the parameter χ and the state of the ablatant with respect to the product $n_e r_p$ at given values of $E_* = 2.0$ keV and $\kappa = 0.29069$. The open marks are results from $r_p = 0.1$ cm. The solid marks are results from $n_e = 5.0 \times 10^{13}$ cm^{-3}.

Risø National Laboratory Risø - M - 2563

Risø - M - 2563

Title and author(s)

SOME OPEN QUESTIONS CONCERNING THE NEUTRAL-SHIELDING MODEL OF A FUELLING PELLET

C.T. Chang

Date February 1986

Department or group

Physics

Group's own registration number(s)

33 pages + 2 tables + illustrations

Abstract

To obtain a better understanding of the implications and assumptions of the idealized neutral shielding model of Parks and Turnbull, the model is reformulated in a self consistent way. Due to the uncertainty of the actual ablation process occuring at the pellet surface, alternative boundary conditions are proposed. Their effect on the pellet ablation rate and the state of the ablated flow are examined by numerical analyses. The results show that the ablation rate is not sensitively affected but the ablatant state is markedly influenced by the boundary condition at the pellet surface. In particular, an increase of the energy flux received at the pellet surface by a factor of four hardly affect the ablation rate but changes the temperature and the density of the ablatant at the pellet surface by three orders of magnitude. Based on these obtained results, it is concluded that the idealized ablation model is adequate when pellet injection is used to fuel a plasma but requires modification when it is used to probe plasma properties and discharge conditions.

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